SCIS-HIS

Math Teaching & Learning Standards
9-12

HS Algebra

Seeing Structure in Expression

- HSA.SSE.1
  Interpret expressions that represent a quantity in terms of its context.[*]
  - HSA.SSE.1.a
    Interpret parts of an expression, such as terms, factors, and coefficients.
  - HSA.SSE.1.b
    Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P.*

- HSA.SSE.2
  Use the structure of an expression to identify ways to rewrite it. *For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.*

- HSA.SSE.3
  Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.[*]
  - HSA.SSE.3.a
    Factor a quadratic expression to reveal the zeros of the function it defines.
  - HSA.SSE.3.b
    Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
  - HSA.SSE.3.c
    Use the properties of exponents to transform expressions for exponential functions. *For example the expression 1.15^t can be rewritten as $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.*
  - HSA.SSE.4
    Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. *For example, calculate mortgage payments.*[*]

HS Algebra (Continued)

Arithmetic with Polynomials & Rational Expressions

▸ HSA.APR.1
Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

▸ HSA.APR.2
Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.

▸ HSA.APR.3
Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

▸ HSA.APR.4
Prove polynomial identities and use them to describe numerical relationships. *For example, the polynomial identity $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples.*

▸ HSA.APR.5
(+) Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.[1]

▸ HSA.APR.6
Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.

▸ HSA.APR.7
(+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.

HS Algebra (Continued)

Creating Equations

▸ HSA.CED.A.1
Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*

▸ HSA.CED.2
Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

▸ HSA.CED.3
Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*

▸ HSA.CED.4
Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law $V = IR$ to highlight resistance R.*

HS Algebra (Continued)

Reasoning with Equations and Inequalities

- HSA.REI.1
  Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

- HSA.REI.2
  Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

- HSA.REI.3
  Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

- HSA.REI.4
  Solve quadratic equations in one variable.
  - HSA.REI.4.a
    Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.
  - HSA.REI.B.4.b
    Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.

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HS Algebra (Continued)

- **HSA.REI.5**
  Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.

- **HSA.REI.6**
  Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

- **HSA.REI.7**
  Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$.

- **HSA.REI.8**
  (+) Represent a system of linear equations as a single matrix equation in a vector variable.

- **HSA.REI.9**
  (+) Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3×3 or greater).

- **HSA.REI.10**
  Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

- **HSA.REI.11**
  Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.[*]

- **HSA.REI.12**
  Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

HS Geometry

Congruence

- HSG.CO.1
  Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

- HSG.CO.2
  Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).

- HSG.CO.3
  Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.

- HSG.CO.4
  Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.

- HSG.CO.5
  Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

- HSG.CO.6
  Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.

- HSG.CO.7
  Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.

- HSG.CO.8
  Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.

HS Geometry (Continued)

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Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.

‣ HSG.CO.9
Prove theorems about lines and angles. *Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.*

‣ HSG.CO.10
Prove theorems about triangles. *Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.*

‣ HSG.CO.11
Prove theorems about parallelograms. *Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.*

HS Geometry (Continued)

- HSG.CO.12
  Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). *Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.*

- HSG.CO.13
  Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

Similarity, Right Triangles, & Trigonometry

- HSG.SRT.1
  Verify experimentally the properties of dilations given by a center and a scale factor:
  - HSG.SRT.1.a
    A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.
  - HSG.SRT.1.b
    The dilation of a line segment is longer or shorter in the ratio given by the scale factor.

- HSG.SRT.2
  Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.

- HSG.SRT.3
  Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.

- HSG.SRT.4
  Prove theorems about triangles. *Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.*

- CHSG.SRT.5
  Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

HS Geometry (Continued)

▸ **CHSG.SRT.6**
Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.

▸ **HSG.SRT.7**
Explain and use the relationship between the sine and cosine of complementary angles.

▸ **HSG.SRT.8**
Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.[*]

▸ **HSG.SRT.9**
(+) Derive the formula $A = 1/2\ ab\ \sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.

▸ **HSG.SRT.10**
(+) Prove the Laws of Sines and Cosines and use them to solve problems.

▸ **HSG.SRT.11**
(+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces)

Circles

▸ **HSG.C.1**
Prove that all circles are similar.

▸ **HSG.C.2**
Identify and describe relationships among inscribed angles, radii, and chords. *Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.*

HS Geometry (Continued)

‣ HSG.C.3
Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.

‣ HSG.C.4
(+) Construct a tangent line from a point outside a given circle to the circle.

‣ HSG.C.5
Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.

Expressing Geometric Properties with Equations

‣ HSG.GPE.1
Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.

‣ HSG.GPE.2
Derive the equation of a parabola given a focus and directrix.

‣ HSG.GPE.3
(+) Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.

‣ HSG.GPE.4
Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, $\sqrt{3}$) lies on the circle centered at the origin and containing the point (0, 2).*

HS Geometry (Continued)

‣ HSG.GPE.5
Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).

‣ HSG.GPE.6
Find the point on a directed line segment between two given points that partitions the segment in a given ratio.

‣ HSG.GPE.7
Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.

Geometric Measurement & Dimension

‣ HSG.GMD.1
Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. *Use dissection arguments, Cavalieri's principle, and informal limit arguments*.

‣ HSG.GMD.2
(+) Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.

‣ HSG.GMD.3
Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.[*]

‣ **Visualize relationships between two-dimensional and three-dimensional objects**

‣ HSG.GMD.4
Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.

HS Statistics and Probability

Interpreting Categorical & Quantitative Data

‣ HSS.ID.1
Represent data with plots on the real number line (dot plots, histograms, and box plots).

‣ HSS.ID.2
Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.

‣ HSS.ID.3
Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).

‣ HSS.ID.4
Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.

‣ HSS.ID.5
Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.

‣ HSS.ID.6
Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.

  ◦ HSS.ID.6.a
  Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.

HS Statistics and Probability (Continued)

- ○ HSS.ID.6.b
  Informally assess the fit of a function by plotting and analyzing residuals.
- ○ HSS.ID.6.c
  Fit a linear function for a scatter plot that suggests a linear association.

▸ HSS.ID.7
Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.

▸ HSS.ID.8
Compute (using technology) and interpret the correlation coefficient of a linear fit.

▸ HSS.ID.9
Distinguish between correlation and causation.

Making Inferences & Justifying Conclusions

▸ HSS.IC.1
Understand statistics as a process for making inferences about population parameters based on a random sample from that population.

▸ HSS.IC.2
Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. *For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?*

▸ HSS.IC.3
Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.

▸ HSS.IC.4
Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.

▸ HSS.IC.5
Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.

▸ HSS.IC.6
Evaluate reports based on data.

HS Statistics and Probability (Continued)

Conditional Probability & the Rules of Probability

▸ HSS.CP.1
Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").

▸ HSS.CP.2
Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.

▸ HSS.CP.3
Understand the conditional probability of A given B as $P(A$ and $B)/P(B)$, and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.

▸ HSS.CP.4
Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.*

▸ HSS.CP.5
Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.*

HS Statistics and Probability (Continued)

- **HSS.CP.6**
  Find the conditional probability of *A* given *B* as the fraction of *B*'s outcomes that also belong to *A*, and interpret the answer in terms of the model.

- **HSS.CP.7**
  Apply the Addition Rule, P(A or B) = P(A) + P(B) − P(A and B), and interpret the answer in terms of the model.

- **HSS.CP.8**
  (+) Apply the general Multiplication Rule in a uniform probability model, P(A and B) = P(A)P(B|A) = P(B)P(A|B), and interpret the answer in terms of the model.

- **HSS.CP.9**
  (+) Use permutations and combinations to compute probabilities of compound events and solve problems.

Using Probability to Make Decisions

- **HSS.MD.1**
  (+) Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions.

- **HSS.MD.2**
  (+) Calculate the expected value of a random variable; interpret it as the mean of the probability distribution.

- **HSS.MD.3**
  (+) Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. *For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.*

HS Statistics and Probability (Continued)

▶ HSS.MD.4
(+) Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. *For example, find a current data distribution on the number of TV sets per household in the United States, and calculate the expected number of sets per household. How many TV sets would you expect to find in 100 randomly selected households?*

▶ HSS.MD.5
(+) Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values.

  ◦ HSS.MD.5.a
  Find the expected payoff for a game of chance. *For example, find the expected winnings from a state lottery ticket or a game at a fast-food restaurant.*

  ◦ HSS.MD.5.b
  Evaluate and compare strategies on the basis of expected values. *For example, compare a high-deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident.*

▶ HSS.MD.6
(+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).

▶ HSS.MD.7
(+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).